Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2014

Question/Answer Booklet

MATHEMATICS: SPECIALIST 3A/3B

Section Two: Calculator-assumed

Please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for this section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

Number of additional answer booklets used (if applicable): ☐

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

 Ref: 14-086

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	33⅓
Section Two: Calculator-assumed	12	12	100	100	66⅔
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2014*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section Two: Calculator-assumed **(100 Marks)**

This section has **12** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 100 minutes.

Question 9 **(6 marks)**

Given that $a^x = 4$ and $a^y = 7$,

(a) evaluate a^{x+y}. (2 marks)

(b) write an expression, in terms of x and/or y, for

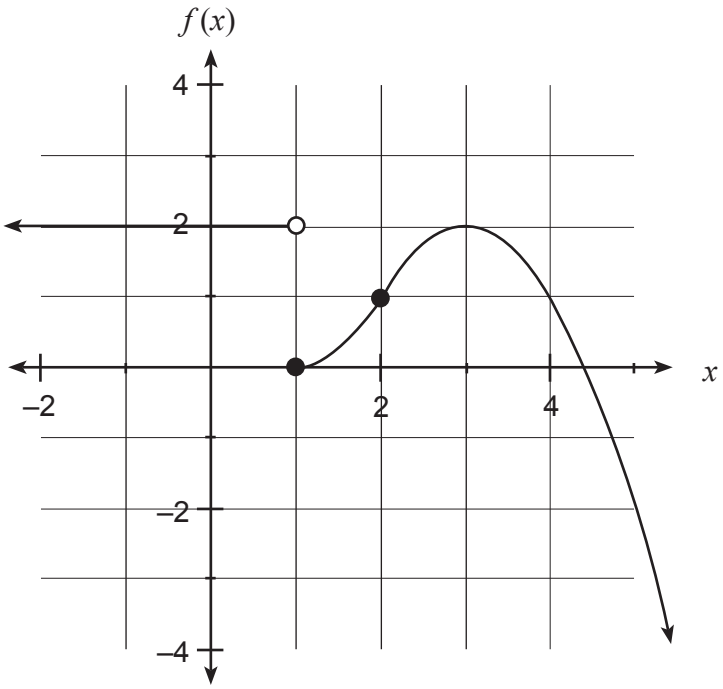
(i) $\log_a\left(\frac{7}{4}\right)$. (2 marks)

(ii) $\log_a (49a^2)$. (2 marks)

Question 10 **(7 marks)**

A piecewise defined function f is defined by $f(x) = \begin{cases} 2 & x < 1 \\ (x-1)^2 & 1 \le x \le 2 \\ 6x - x^2 - 7 & x > 2 \end{cases}$.

The graph of this function is shown below.



(a) Explain why the function f is not differentiable at $x = 1$. (1 mark)

(b) Evaluate

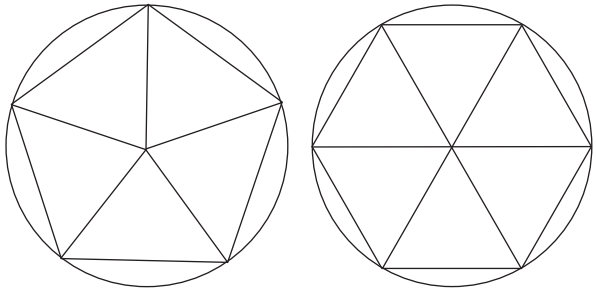
(i) $\lim_{x \to 2^-} f'(x)$. (2 marks)

(ii) $\lim_{x \to 2^+} f'(x)$. (2 marks)

(c) Is the function f differentiable at $x = 2$? Justify your answer. (2 marks)

Question 11 **(5 marks)**

The diagrams below show examples of a regular n-sided polygon inscribed in a circle of radius 1 unit.



$n = 5$ $n = 6$

Let $A(n)$ = the area of the inscribed regular n-sided polygon.

n	Expression for $A(n)$	Polygon area (in square units to four decimal places)
4	$4 \times \frac{1}{2} \times \sin\left(\frac{2\pi}{4}\right)$	2.0000
5	$5 \times \frac{1}{2} \times \sin\left(\frac{2\pi}{5}\right)$	2.3776
6	$6 \times \frac{1}{2} \times \sin\left(\frac{2\pi}{6}\right)$	
10		2.9389

(a) Evaluate $A(6)$ correct to four decimal places. (1 mark)

(b) Write the expression for $A(10)$, the area for a polygon with 10 sides. (1 mark)

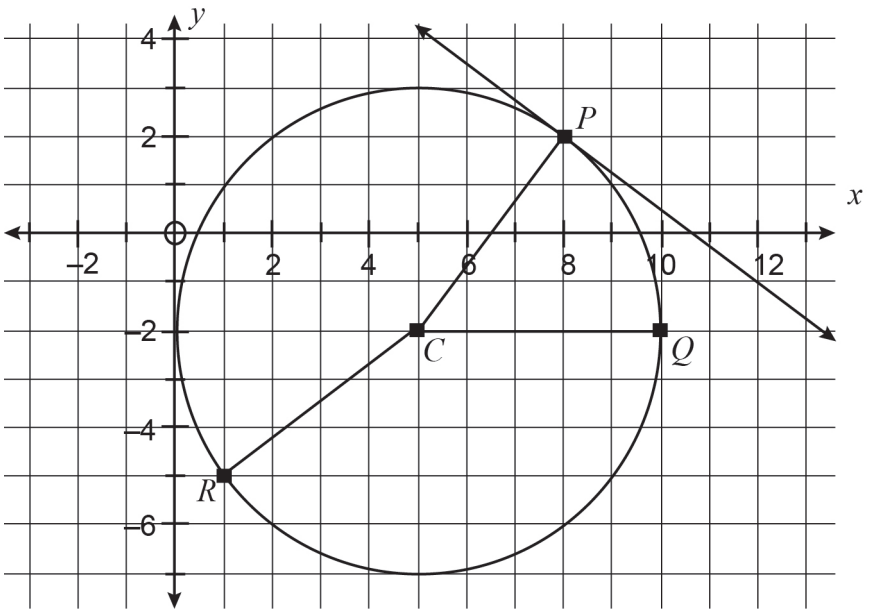
(c) Suggest the expression for $A(n)$, the area for a polygon with n sides. (1 mark)

(d) Determine the exact limiting value of $A(n)$ as $n \to \infty$. (2 marks)
Explain why this value occurs.

See next page

Question 12 **(17 marks)**

A circle with centre point C has vector equation $\left|\mathbf{r} - \begin{pmatrix} 5 \\ -2 \end{pmatrix}\right| = 5$. A tangent to the circle is drawn at the point P, which has position vector $\begin{pmatrix} 8 \\ 2 \end{pmatrix}$.



(a) Determine the

(i) distance CP. (1 mark)

(ii) vector $\overrightarrow{CP}$. (2 marks)

(b) Point R has position vector $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$. Determine the

(i) size of the obtuse angle $\angle RCP$ correct to the nearest degree. (3 marks)

See next page

(ii) vector equation for the tangent at point P. (3 marks)

(c) Point Q has position vector $\begin{pmatrix} 10 \\ -2 \end{pmatrix}$. Determine the

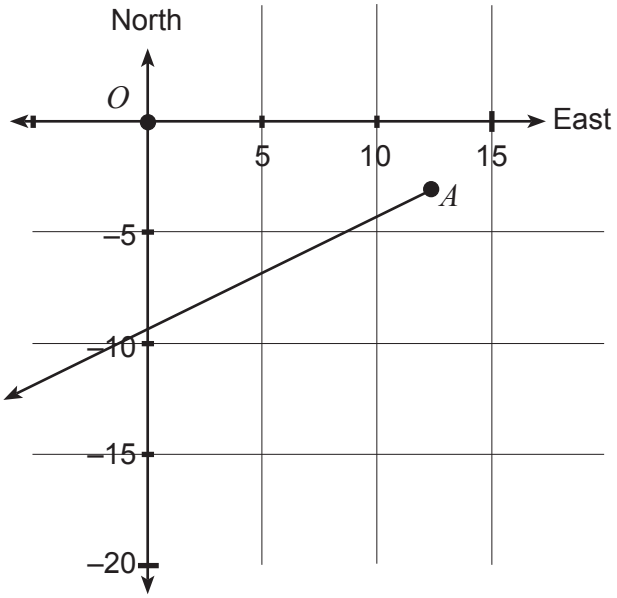
(i) area of minor sector PCQ, correct to 0.01 square units. (4 marks)

(ii) exact position vectors of both of the x-intercepts of the circle. (4 marks)

See next page

Question 13 **(10 marks)**

At 0900 hours, a boat is positioned at point A at $12.4\mathbf{i} - 3.1\mathbf{j}$ km relative to a communications base at the origin O. The boat moves with a constant velocity of $-7\mathbf{i} - 3.5\mathbf{j}$ km/h along the path shown below.



(a) Determine the initial bearing of the boat from the communications base, correct to the nearest degree. (2 marks)

(b) Determine the speed of the boat, correct to the nearest 0.01 km/h. (1 mark)

(c) Determine when the boat will be directly south of the communications base. (3 marks)

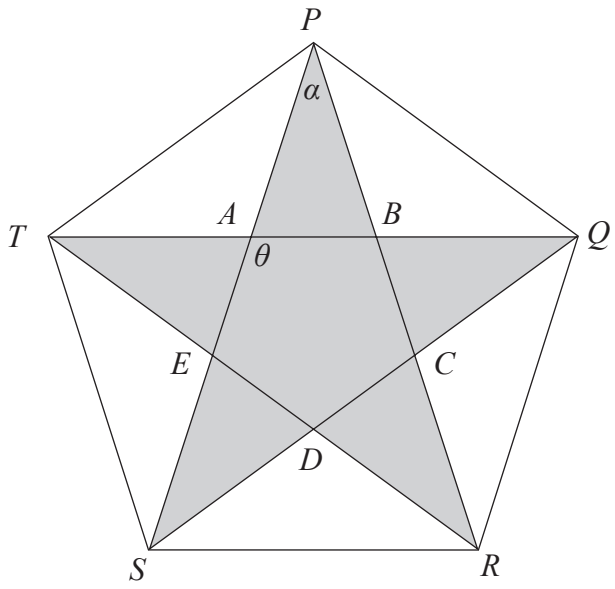
(d) Determine the shortest distance between the boat and the communications base, correct to the nearest 100 metres. (4 marks)

See next page

Question 14 **(7 marks)**

The diagram below shows a shaded five-pointed star, called a pentagram. This pentagram has the following properties:

- pentagon $ABCDE$ is regular i.e. all sides and angles are congruent
- ΔPAB is isosceles, with $PA = PB$
- length $AB = 10$ cm
- T, A, B and Q are collinear points.



It can be shown that $\theta = s\angle EAB = 108º$ and $\alpha = s\angle APB = 36º$.

Calculate, correct to three decimal places, the

(a) length AP. (2 marks)

See next page

(b) length PT. (2 marks)

(c) ratio of the area of ΔPAT to the area of ΔPAB .
i.e. calculate $\frac{\text{Area } \Delta PAT}{\text{Area } \Delta PAB}$. (3 marks)

See next page

Question 15 **(12 marks)**

The loudness of a sound L, measured in decibels (dB), is given by:

$$L = 10 \log\left(\frac{I}{I_0}\right)$$

where I = the intensity of a sound (in watts per square metre) and
where I_0 = the intensity of the threshold of hearing (in watts per square metre).

The threshold of hearing is the intensity that is just audible to the human ear. It is known that $I_0 = 10^{-12}$ watts per square metre.

(a) Determine the loudness of a sound, measured in decibels, that is at the threshold of hearing. (2 marks)

(b) If the intensity of a sound is doubled from $I = I_1$ to $I = 2I_1$, determine the increase in the loudness of that sound, correct to the nearest decibel. (3 marks)

See next page

At a house party, an amplifier is set at a level so that it is radiating 5 watts of sound power from the speakers. It is found that the sound intensity, I, of this music varies with the distance d from the speakers according to the relationship:

$$I = \frac{5}{\pi d^3}$$

where d = the distance from the speakers in metres.

(c) Calculate the loudness of the party music at a distance of 10 metres, correct to the nearest decibel. (2 marks)

(d) If the loudness L is written in the general form $L = a - b\,(\log d)$, determine the values of the constants a and b. (3 marks)

The party music is considered to be at an acceptable level for the neighbours if the loudness level is below conversation level (60 dB).

(e) At what distance, correct to the nearest metre, will the party music be considered to be acceptable? (2 marks)

See next page

Question 16 **(8 marks)**

Consider the complex numbers given by $z_1 = 2 + 3i$ and $z_2 = 1 - i$.

(a) Determine the exact value of $\overline{z_1 z_2}$. (2 marks)

(b) Determine the complex number z_3 such that $z_1 = \frac{z_2}{z_3}$. (2 marks)

(c) Determine the quadratic equation $z^2 + bz + c = 0$ having solutions $z = z_2$ and $z = \overline{z_2}$. (4 marks)

Question 17 **(6 marks)**

Assume that the earth is a sphere of radius 6370 km.

(a) Calculate the circumference of the line of latitude 32°S, correct to the nearest kilometre. (2 marks)

The latitude and longitude of Perth and Taree (a town on the mid-north coast of New South Wales) are shown in the following table:

Location	Latitude	Longitude
Perth	32°S	116°E
Taree	32°S	152°E

(b) What is the distance from Perth to Taree along the line of latitude 32°S? (2 marks)

(c) A friend tells you that this is **not** the shortest distance between Perth and Taree, when travelling on the earth's surface. Explain why your friend is correct. (2 marks)

See next page

Question 18 **(7 marks)**

A botanist has found that the rate of the loss of leaves from a particular rainforest tree is proportional to the number of leaves on the tree.

If the initial number of leaves on the tree is L_0 , then the number of these leaves that remain after time t years is

$$L(t) = L_0 e^{-kt}$$

where k is a positive constant.

(a) The botanist wishes to study lichens that grow on the leaves of this tree. From past observations, he has found that of 100 leaves tagged, 80 remain after three years. Use this information to determine the value of the constant k. (2 marks)

(b) Using your result from part (a), calculate the number of leaves that the botanist should initially tag if he needs to have 200 leaves to study in eight years' time. (3 marks)

See next page

(c) If initially N leaves are tagged, calculate when only half of them will remain on the tree. Give your answer correct to two significant figures. (2 marks)

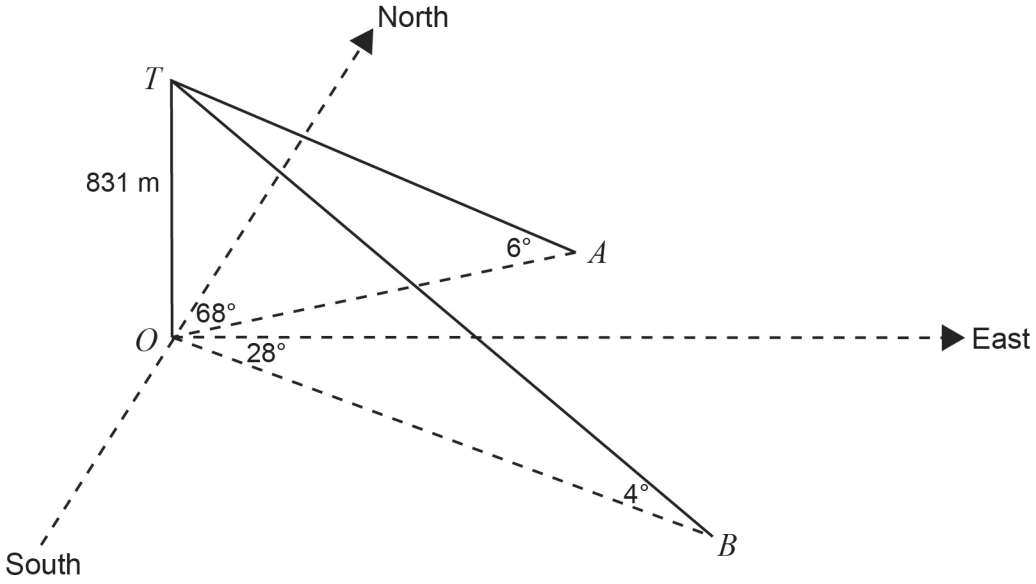
See next page

Question 19 **(9 marks)**

The top of a mountain, T, is 831 m vertically above a flat plain. From point O, the base of the mountain, point A is on a bearing of 068°, while point B is on a bearing of 118°.

The top of the mountain, T, is observed to be at an angle of elevation of 6° from point A, while it is 4° from B.

Some of this information is shown in the diagram below (not drawn to scale):



(a) Calculate, correct to the nearest metre,

(i) the distance OA. (2 marks)

(ii) the distance OB. (1 mark)

(b) Hence, calculate the distance AB, correct to the nearest metre. (3 marks)

(c) Determine the bearing of point A from point B, correct to the nearest degree. (3 marks)

See next page

Question 20 **(6 marks)**

(a) Prove the identity $\cos(4\theta) = 8\cos^4\theta - 8\cos^2\theta + 1$. (3 marks)

(b) The value $x = \cos\left(\frac{\pi}{12}\right)$ is a solution to the equation $ax^4 + bx^2 + 1 = 0$.

Determine possible values of the constants a and b. (3 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____

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